Midterm is on October 12, in AUST 110 - austin building by the lake
IN PERSON, OPEN NOTES, NO TEXTBOOK, NO DEVICES

HOMEWORK 5, PROBLEM 2

a) This is a set of pairs of the form \((course, prof)\), where \(course \in A\) and \(prof \in B\).
   - teachingNow\((course, prof)\): The prof is teaching this course now
   - hasTaught\((course, prof)\): the prof has taught this course at some point
   - canTeach\((course, prof)\): the prof is qualified to teach this course

b) This is a set of TRIPLES of the form \((airline, start, dest)\), where \(airline \in A\), \(start \in B\), \(dest \in C\)
   - directFlight\((airline, start, dest)\): airline offers a direct flight from start to dest
   - affordableFlight\((airline, start, dest)\): airline can get you from start to dest for < $200

HOMEWORK 5, PROBLEM 3

If \(|S| = n\), then \(|P(S)| = 2^n\)

a) \(P\{\"n\}\} = \{\emptyset, \{\"n\}\}\)

b) \(P\{\"n\", \"m\", \"k\}\} = \{\emptyset, \{\"n\\}, \{\"m\}\}, \{\"k\}\}, \{\"n\", \"m\}\}, \{\"n\", \"k\}\}, \{\"m\", \"k\}\}, \{\"n\", \"m\", \"k\}\}\}

c) \(P\{\emptyset, \{\emptyset\}\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}

d) \(P\{\"n\", \"m\}\} \times \{\"m\", \"k\}\} = \{\{\"n\", \"m\}\}, \{\"n\", \"k\}\}, \{\"m\", \"m\}\}, \{\"m\", \"k\}\}\} = \{a,b,c,d\}

Let \(a = \{\"n\", \"m\}\}, \text{ let } b = \{\"n\", \"k\}\}, \text{ let } c = \{\"m\", \"m\}\}, \text{ let } d = \{\"m\", \"k\}\}

\(P\{\"n\", \"m\}\} \times \{\"m\", \"k\}\} = P\{a,b,c,d\} = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}

HOMEWORK 5, PROBLEM 4

\(R\) is SYMMETRIC: for all \(a,b\) in \(D\), \((a,b) \in R \rightarrow (b,a) \in R\) (definition)

\(R\) is REFLEXIVE: for all \(x\) in \(D\) : \((x, x) \in R\) (definition)

\(R\) is TRANSITIVE: FOR ALL \(a, b, c\) in \(D\), whenever \((a,b) \in R\) and \((b,c) \in R\), then \((a,c) \in R\) (definition)
a) HasSameLength
1. Let \( w, u, z \) be any strings
2. It is always the case that HasSameLength\((w, w)\)
3. Therefore by definition, HasSameLength is reflexive
4. Whenever HasSameLength\((w, u)\) it is also the case that HasSameLength\((u, w)\).
5. Therefore by definition, HasSameLength is symmetric.
6. Whenever HasSameLength\((w, u)\) and HasSameLength\((u, z)\), it is also the case HasSameLength\((w, z)\)
7. Therefore by definition, HasSameLength is transitive
8. Since we've shown that HasSameLength is reflexive, symmetric, and transitive, then by definition it is an equivalence relation

b) AtLeastAsLong
1. Consider strings "hi" and "hello" (COUNTEREXAMPLE)
2. It is the case that "hello" is at least as long as "hi", but not vice versa.
3. Thefore \("hello", "hi" \) \(\in\) AtLeastAsLong, but \("hi", "hello" \) \(\not\in\) AtLeastAsLong
4. By definition, this means that AtLeastAsLong is not symmetric.
5. Therefore, according to definition of an equivalence relation, AtLeastAsLong is not an equivalence relation

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**HOMEWORK 5, PROBLEM 5**

a) \( \forall a, b: (a \neq b) \rightarrow !(a R b) \lor !(b R a) \)

CONTRAPOSITIVE: \( \forall a, b: !(!(a R b) \lor !(b R a)) \rightarrow !(a \neq b) \)

SIMPLIFY: \( (a R b) \land (b R a) \rightarrow (a = b) \)

REMINDER: \( x \rightarrow y \) CONTRAPOSITIVE \( !y \rightarrow !x \)

b) AtLeastAsLong is not a antisymmetric
   1. Consider any two strings \( w, u \) where \( w \neq u \)
   2. We know that you CAN have both AtLeastAsLong\((w,u)\) and AtLeastAsLong\((u,w)\) if these are two different strings of the same length
   3. Therefore, AtLeastAsLong is NOT antisymmetric

d) LongerThan is not a partial order
   1. Consider any string \( w \)
   2. It is not true that LongerThan\((w,w)\), because a string cannot be longer than itself.
   3 Therefore, LongerThan is not reflexive
   4. Therefore, LongerThan is not a partial order
e) SameString is both an equivalence relation and a partial order reflexive, symmetric, antisymmetric, and transitive

HOMEWORK 5, PROBLEM 6

Let $S = \{1,2,3,4,5,6\}$

a)
1. Let $S' = \{\{1\}, \{2,3\}, \{4, 5, 6\}\}$. $S'$ consists of non-empty subsets of $S$.
2. $\{1\} \cup \{2,3\} \cup \{4, 5, 6\} = \{1,2,3,4,5,6\} = S$
3. $\{1\} \cap \{2,3\} = \{\} \cap \{4, 5, 6\} = \{1\} \cap \{4,5,6\} = \emptyset$
4. Since both conditions are satisfied, it follows by definition that $S'$ is a partition of $S$

b)
1. Let $S' = \{\{1\}, \{2,3\}, \{4, 5\}\}$
2. Note that $\{1\} \cup \{2,3\} \cup \{4, 5\} = \{1,2,3,4,5\} \neq S$ (6 is missing)
3. Since the union of all sets in $S'$ does not equal $S$, then by definition, $S'$ is not a partition of $S$

c)
1. Let $S' = \{\{1\}, \{2,3,4\}, \{4,5,6\}\}$
2. Note that $\{2,3,4\} \cap \{4,5,6\} = \{4\} \neq \emptyset$
3. Since there is a pair in $S'$ whose intersection is not empty, by definition, $S'$ is not a partition of $S$

S - finite strings
$S_k$ denotes the set of all strings in $S$ of length $k$
1. $S' = \{S_0, S_1, S_2, S_3, ...\}$ - a set of non-empty subsets of $S$
2. Consider an arbitrary string $w$ of length $n$, where $n$ is a natural number $\geq 0$
3. Then $w \in S_n$
4. Consider $S_0 \cup S_1 \cup S_2 \cup S_3 \cup ...$. It must include $w$ because $S_n$ is a subset of this union
5. Therefore $S' = S_0 \cup S_1 \cup S_2 \cup S_3 \cup ...$ (the union of all sets in $S'$ equals $S$)
6. Let $S_i$ and $S_j$ be two distinct members of $S'$ (where $i \neq j$)
7. Their intersection must be empty, since a string cannot be of length $i$ and $j$ at the same time
8. We have shown that the union of all sets in $S'$ equals $S$ and that the sets in $S'$ are all pairwise disjoint. Therefore, by definition, $S'$ is a partition of $S$. 
HOMEWORK 2, PROBLEM 2

a) It snows whenever the wind blows from the Northeast
   if windBlowsFromNortheast then itSnows

b) The apple tree will blossom if it stays warm for a week
   if staysWarmForWeek then appleTreeWillBlossom

c) It is necessary to walk 8 miles to get to the top of Long's peak
   if not walk8miles then not getTopLongsPeak
   if getTopLongsPeak then walk8miles (contrapositive)

d) To get tenure as a professor, it is sufficient to be world-famous
   if worldFamous then getTenure

e) Your guarantee is good only if you bought your CD player less than 90 days ago
   if !boughtCDplayerLessThan90days then !guaranteeGood
contrapositive:
   if guaranteeGood then boughtCDplayerLessThan90days

MORE REVIEW

For all x in D, (x, x) is in R
ALSO WRITTEN AS R(x, x)  ALSO WRITTEN AS x R x

Given R = ............
1. Let a be any member of D
2. Show that (a,a) is in R
3. Therefore, for all a in D, (a,a) is in R
4. By definition this means that R is reflexive

For all x,y in D, if (x, y) is in R then (y,x) is in R
1. Let a,b be any members of D
2. Assume that (a,b) is in R; show that (b, a) must also be in R
3. By definition this means that R is symmetric

For all x,y in D, if R(x, y) and R(y,x) then x = y
1. Let a,b be any members of D
2. Assume that (a,b) is in R and (b, a) is in R; show that a must equal b
3. By definition this means that R is antisymmetric