Problem 1: Tertiary cartesian products

As we know, \( S_1 \times S_2 \times S_3 = \{(x_1, x_2, x_3): x_1 \in S_1 \land x_2 \in S_2 \land x_3 \in S_3\} \)

a) Let \( A = \{a, b, c\}, B = \{x, y\}, C = \{0, 1\}. \) Find the following:

1. \( A \times B \times C \)
2. \( C \times B \times A \)
3. \( B \times B \times B \)
4. \( A \times (B \times C) \) Hint: this is the same as \( A \times D \) where \( D = B \times C \)

b) Suppose that for some sets \( A, B, C, \) it is known that \( A \times B \times C = \emptyset. \) What can you conclude from this? *Prove* your conclusion, using definitions.

Problem 2: Cartesian products and truth sets

a) What is the cartesian product \( A \times B, \) where \( A \) is the set of courses offered by the CSE department, and \( B \) is the set of professors at the CSE department? What binary predicate could you define whose truth set is a subset of \( A \times B? \) Use English for your answers...

b) What is the cartesian product \( A \times B \times C \) where \( A \) is the set of all airlines and \( B \) and \( C \) are both the set of all cities in the United States? What 3-argument predicate could you define whose truth set is a subset of \( A \times B \times C? \) Use English for your answers...

Problem 3: Power sets

Find the power set of each of the following sets:

a) \("n"\)

b) \("n", "m", "k"\)

c) \(\emptyset, \{\emptyset\}\)

d) \("n", "m"\) \times \("m", "k"\)
Problem 4: Equivalence Relations

Definition: an equivalence relation as a relation that is reflexive, symmetric, and transitive (i.e. has all three properties). Using the definitions of these properties, show that:

a) the relation HasSameLength over strings is an equivalence relation
b) the relation AtLeastAsLong over strings is not an equivalence relation

Problem 5: Partial Orders

Definition: We say that a relation \( R \) is antisymmetric if, for any elements \( a, b \), it is not the case that both \( a \ R \ b \) and \( b \ R \ a \), unless \( a=b \).

a) Restate the definition above using quantifiers and logic operations.
b) Using definitions, show that AtLeastAsLong is antisymmetric

definition: a partial order is a relation that is reflexive, antisymmetric, and transitive (i.e. has all three properties). Using the definitions of these properties, show that:

c) AtLeastAsLong is a partial order
d) LongerThan is not a partial order
e) SameString is both an equivalence relation and a partial order

Problem 6: Partitions

A partition of a set \( S \) is defined as a subset \( S' \) of \( P(S) \) (power set of \( S \)) such that:

1. The union of all sets in \( S' \) equals \( S \)
2. For any two set in \( S' \), their intersection is empty

Using this definition, show the following:

a) \( \{\{1\}, \{2,3\}, \{4, 5, 6\}\} \) is a partition of \( \{1,2,3,4,5,6\} \)
b) \( \{\{1\}, \{2,3\}, \{4, 5\}\} \) is not a partition of \( \{1,2,3,4,5,6\} \)
c) \( \{\{1,2\}, \{2,3,4\}, \{5,6\}\} \) is not a partition of \( \{1,2,3,4,5,6\} \)

d) Using definitions, show that \( S' \) is a partition of \( S \).