Please read the note as well as textbook sections for Lectures 23 and 24 before doing homework.

Problem 1

In this problem, we assume that the chance of a couple having a boy is 0.6 and of having a girl is 0.4? (In reality, it's closer to .51 and .49)

a) If a couple has 5 children, what is the chance that 3 of them are boys?

Hint: the chance of having 3 boys is the same no matter what order they come in. How many different combinations of 5 children with 3 boys are there? What is the probability of each one?

b) If a couple has 5 children, what is the chance that they are all the same gender? (i.e. either all girls or all boys)

c) If a couple has 5 children, what is the chance that none of them is a boy?

d) If a couple has 5 children, what is the chance that at least one of them is a girl?

Hint: what is the chance that none of them are girls?

Problem 2

a-d) These are the same questions are above, but now we have k kids instead of 5.

Turn answers from Problem 1 a-d into formulas that are dependent on k. Note that if we assume that \( C(n,m) = 0 \) when \( m > n \), then your formulas should work for all k. To check your answers, plug in some values for k, especially k=5.

e) How many kids should a family plan to have so the chance of having at least one girl is \( \geq 90\% \)?

Hint: you are looking for smallest k so the value of the corresponding formula is \( \geq 90\% \)

Problem 3

For any pregnancy, a couple has 10% chance of having twins, and 90% chance of a single child.

a) Draw the probability tree for three pregnancies. At each edge, show the corresponding probability (either .1 or .9) and at each leaf, the total number of children.

Hint: at the root, the number is 0.

b) Using your tree, what is the probability of having exactly 3 children after 3 pregnancies? Show your computation!

c) Check yourself by computing the same thing as (b) using the methods from problems 1-2 instead. Show your computation!

d) Using your tree, what is the probability of having at least 4 children after 3 pregnancies?
**Problem 4**

We know that events A and B are independent iff $P(A \text{ and } B) = P(A) * P(B)$. 

Prove that if A and B are independent, and C is the complement of B, then A and C are also independent. 

Hint: Use the fact that $P(B) + P(C) = 1$.

**Problem 5**

There are three types of people: 80% are healthy (H), 15% are unhealthy (U), and 5% are very unhealthy (V). There is also a condition C. In a population of 100K, C is found in 5K people in each group.

a) Draw a table of type vs. presence of C, filling in each cell with the number of people. 

Hint: it should have 6 cells, and the numbers is all cells should add up to 100K.

b) For each type, if we take a random person of that type, what is their probability of having C? Compute the three values. 

Note: these are conditional probabilities $P(C | H)$, $P(C | U)$, $P(C | V)$.

c) Represent your scenario as a probability tree 

Hint: the root should have 3 children, whose probabilities add up to 1.

d) Use your tree to compute the probability of a random person not having C. 

Hint: there should be 3 leaves of interest in your tree.

**Problem 6**

There are three types of people: 40% are young (Y), 50% are midde-age (M), and 10% are old (O). There is also a disease D, that has afflicted 1% of the young, 10% of the middle-aged, and 80% of the old.

a) For a population of 100K, draw a table of type vs. presence of disease, filling in each cell with the number of people. 

Hint: it should have 6 cells, and the numbers is all cells should add up to 100K.

b) What is $P(D)$? Meaning, if we take a random person, what is their probability of being sick? 

c) What are $P(Y \cap D)$, $P(M \cap D)$, $P(O \cap D)$? 

Hint: your table should make b and c easy to answer

d) Compute the conditional probabilities $P(Y | D)$, $P(M | D)$, $P(O | D)$. 

Hint: use the formula from the definition of conditional probability, together with your answer to (c)

e) Use all the numbers you have so far to confirm that for each type of person T (T is Y, M, or O), the following is true: $P(T | D) * P(D) = P(D | T) * P(T) = P(T \cap D)$