Please read the textbook sections for Lectures 21 and 22 before doing homework.
Whenever appropriate, show the size of the sample space S and the size of the event space E

Problem 1
a) Given a random permutation of the list \{1,2,3\} what is the chance that 3 precedes 1?
b) Given a random permutation of the list \{1,2,3\} what is the chance that 3 precedes both 1 and 2?

Problem 2
In this problem, we assume that a couple is equally likely to have a boy or a girl as a child.
a) If a couple has 5 children, what is the chance that 3 of them are boys?
b) If a couple has 5 children, what is the chance that none of them are boys?
c) If a couple has 5 children, what is the chance that they are all the same gender? (i.e. either all girls or all boys)
d) How many couples of 5 children should we choose to be guaranteed that two of them have the same number of boys?

Problem 3
a) Given a string consisting of 26 characters ‘a’..’z’ (no repetitions) in some order , what is the probability that ‘a’ and ‘z’ are next to each other? (i.e. “az” or “za”)
   Hint: to find the size of E, can we compute the number of possible strings if we treat “az” as a special character that replaces ‘a’ and ‘z’?
b) Given a string consisting of 26 characters ‘a’..’z’ (no repetitions) in some order , what is the probability that it contains 13 characters IN ORDER? (i.e. “abcde…”, or “bcdef…n”, or “nmopq…z” etc)
   Hint: can we do something similar to what we did for (a)?
c) Given a string consisting of 26 characters ‘a’..’z’ (no repetitions) in some order , what is the probability that ‘a’ and ‘b’ are not next to each other?
   Hint: consider the event when they are next to each other; have we seen a similar problem before?
d) Given a string consisting of 26 characters ‘a’..’z’ (no repetitions) in some order , what is the probability that that ‘a’ and ‘z’ are separated by either 23 or 24 letters?
   Hint: to find the size of E, first see how many ways there are to place ‘a’ and ‘z’ into the string, then fill in the rest with remaining letters.
e) Given a string consisting of 26 characters ‘a’..’z’ (no repetitions) in some order , what is the probability that ‘z’ precedes both ‘a’ and ‘b’?
   Hint: you not have to compute the size of E, if you focus on swapping around the letters ‘a’, ‘b’, and ‘z’
Problem 4

Suppose we have two events E1 and E2 over the same space S. We know that \( P(E1) = 0.8 \) and \( P(E2) = 0.6 \). We know the following facts, for any events E and F:

1. If E is a subset of F, then \( N(E) \leq N(S) \)
2. \( N(E \cup F) = N(E) + N(F) - N(E \cap F) \)
3. Probability of E = \( P(E) = \frac{N(E)}{N(S)} \)

Use this information to prove that \( P(E1 \cap E2) \geq 0.4 \) and \( P(E1 \cup E2) \geq 0.8 \)

Problem 5

a) If we choose two people at random, what is the probability that they have their birthday on the same day of the week?

b) If we have 7 people in a group, what is the probability that none of them have a birthday on the same day of the week?

c) If we have 7 people in a group, what is the probability that at least two of them have a birthday on the same day of the week?

Hint: use the answer from (b)

d) How many people should there be in a group, so the chance of having two of them with a birthday on the same day of the week is \( \geq 0.5 \) ?

e) How many people should there be in a group, to be guaranteed that at least two of them have a birthday on the same day of the week?

Hint: Pigeonhole Principle

Problem 6

a) Consider a group on n people. What is the probability that none of them have a birthday today?

Hint: think of their birthdays as an n-tuple of values (1…365), one value for each date of the year; we are interested in those tuples that do not contain the value that matches today’s date.

b) Consider a group on n people. What is the probability that at least one of them has a birthday today?

Hint: use your answer from (a)

c) What is the smallest possible group of people, so the chance of at least one of them having a birthday today is \( \geq 0.5 \) ?