Reading (in textbook, unless stated otherwise):

• Set operations: Section 6.1 from middle of p. 381 through example 6.1.5  
• Reread 2.1 and 2.2

NOTE: please study examples carefully, there are many more of them than we possibly have time for in class

REVIEW

Doing proof is like climbing a wall

1. plan the climb - pick out the steps in any order, until they all connect
2. execute - must be in order

Definitions – to be used in proofs

EXAMPLE: a number is even if it is divisible by 2

USED IN BOTH DIRECTIONS

if you see an even number, can conclude it's divisible by 2
if you see a number divisible by 2, can conclude it's even

"plugging in" – need to do it when using definitions

EXAMPLE:

definition of "x is even": x is divisible by 2
Question: is 7 even?
plug in 7 for x!!
“7 is even”: 7 is divisible by 2
this is false, so 7 is not even

TEXTBOOK: tons of examples, read it!!

Using constants (TRUE, FALSE) in propositional formulae

\[
\begin{align*}
A \lor \text{TRUE} & \equiv \text{TRUE} \\
A \land \text{TRUE} & \equiv A \\
A \lor \text{FALSE} & \equiv A \\
A \land \text{FALSE} & \equiv \text{FALSE}
\end{align*}
\]

Shown with truth tables

\[
\begin{array}{ccc}
A & A \lor \text{FALSE} & A \land \text{FALSE} \\
\text{TRUE} & \text{FALSE} & \text{TRUE} \\
\text{FALSE} & \text{FALSE} & \text{FALSE}
\end{array}
\]
Translating from English to logic

English has many ways to say same thing
conversely, the same thing in English can have multiple meanings in logic (we use context to disambiguate)

EXAMPLE: do you want milk or juice (XOR = choose one, OR = choose at least one)

SET UNION ∪

It's a set operation, takes two sets and creates a new one

DEFINITION: Given sets S1 and S2, \( S1 ∪ S2 = \{x : x ∈ S1 ∨ x ∈ S2\} \)
“all x such that x is a member of S1 OR x is a member of S2”

EXAMPLE: \( S1 = \{a, b\}, S2 = \{b, c\} \), then \( S1 ∪ S2 = \{a, b, c\} \)

SET INTERSECTION ∩

Also a set operation, takes two sets and creates a new one

DEFINITION: Given sets S1 and S2, \( S1 ∩ S2 = \{x : x ∈ S1 ∧ x ∈ S2\} \)
“all x such that x is a member of S1 AND x is a member of S2”

EXAMPLE: \( S1 = \{a, b\}, S2 = \{b, c\} \), then \( S1 ∩ S2 = \{b\} \)

EXAMPLE: \( E = \{x ∈ Z : x \text{ is even}\}, G = \{x ∈ Z : x > 100\} \)

\[ E ∪ G = \{x ∈ Z : x \text{ is even OR } x > 100\} \]
\[ E ∩ G = \{x ∈ Z : x \text{ is even AND } x > 100\} = \{102, 104, 106 \ldots\} \]

EXAMPLE: \( D = \{x : x \text{ is a string starting with 'd'}\}, I = \{x : x \text{ is a string where 2nd char is 'i'}\} \)

\( D ∪ I = \{x : x \text{ is a string where the first char is 'd' OR 2nd char is 'i'}\} \)
\{“dina”, “double”, “bimbo” …\}

\( D ∩ I = \{x : x \text{ is a string where 1st char is 'd' AND 2nd char is 'i'}\} = \{x : \text{all strings that start with “di”}\} \)
\{“die”, “dina”, “divine” …\}

Determining if a value is a member of set union or intersection

EXAMPLE: is “bob” a member of \( D ∪ I \) ?

1. By definition, this is the same as asking whether “bob” ∈D ∨ “bob” ∈I
2. “bob” does not belong to D because it does not start with 'd'
3. “bob” does not belong to I because 2nd char is not 'i'
4. So it is not true that “bob” ∈D ∨ “bob” ∈I
5. Therefore, by definition, b does not belong to \( D ∪ I \)

SET DIFFERENCE –
It's another set operation, takes two sets and creates a new one

**DEFINITION:** Given sets $S_1$ and $S_2$, $S_1 - S_2 = \{x : x \in S_1 \land x \not\in S_2\}$

“all $x$ such that $x$ is a member of $S_1$ OR $x$ is a member of $S_2$”

**EXAMPLE:** $S_1 = \{a, b\}$, $S_2 = \{b, c\}$, then $S_1 - S_2 = \{a\}$, $S_2 - S_1 = \{c\}$

**EXAMPLE:** Using $D$ and $I$ as above, $D - I =$ strings starting with 'd' EXCEPT for those whose 2nd char is 'l'

**Visualizing** the three operations UNION, INTERSECTION, DIFFERENCE

Claim: **for any set** $S$, $S \cup \emptyset = S$

This is an example of how we **plug in** when we use definitions

**Proof:**
1. by definition of $\cup$, $S \cup \emptyset = \{x : x \in S \lor x \in \emptyset\}$
2. by definition of $\emptyset$, $x \in \emptyset$ is FALSE for all $x$
3. therefore $S \cup \emptyset = \{x : x \in S \lor x \in \emptyset\} = \{x : x \in S \lor \text{FALSE}\} = \{x : x \in S\} = S$

QED

Claim: **for any set** $S$, $S \cap \emptyset = \emptyset$

Proven similarly

Note that we get $\emptyset$ whenever we take intersection of two sets whose membership properties are incompatible

**EXAMPLE:** Evens = \{x \in \mathbb{Z} : x \text{ is even}\}, Odds = \{x \in \mathbb{Z} : x \text{ is odd}\}

Evens $\cup$ Odds = $\mathbb{Z}$
Evens $\cap$ Odds = \{x \in \mathbb{Z} : \text{x is even AND x is odd}\} = $\emptyset$ since an integer cannot be both even and odd

**Multiple arguments for AND, OR**

$X \lor Y \lor Z$ TRUE WHEN "at least one is true" = FALSE when ALL are false
$X \land Y \land Z$ TRUE WHEN "all are true" = FALSE when "at least one is false"

**Associative property** – order of evaluation does not matter
EXAMPLE: + is associative \((a + b) + c = a + (b + c)\)

EXAMPLE: - is not associative \((a - b) - c \neq a - (b - c)\)

AND, OR are associative (as well as commutative)

\[
\begin{align*}
X \lor Y \lor Z &= (X \lor Y) \lor Z = X \lor (Y \lor Z) \\
X \land Y \land Z &= (X \land Y) \land Z = X \land (Y \land Z)
\end{align*}
\]

proven with truth tables

IMPLICATION is not associative (proven in homework)

\[
(X \rightarrow Y) \rightarrow Z \neq X \rightarrow (Y \rightarrow Z)
\]