Reading (in textbook, unless stated otherwise):

- Chapter 2.1
- Chapter 2.2 through example 2.2.4

NOTE: please study examples carefully, there are many more of them than we possibly have time for in class

PROPOSITIONAL LOGIC

**Proposition** – a statement that is true or false

Such as “facts” for proofs – known to be true; conclusions also – you have to show it’s true

**Examples:**

- 3 is odd
- S1 = …
- S1 ⊆ S2

**Propositional Operators** – apply to propositions to get new ones

**Negation** – unary operator, \( ! \), \( \sim \)

\( !p \) – new proposition

TRUTH TABLES

# of rows = # of combinations of values for all variables (no particular order needed)
# of columns = one for each variable, one for each operator

**Truth table** for negation

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \neg p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRUE</td>
<td>FALSE</td>
</tr>
<tr>
<td>FALSE</td>
<td>TRUE</td>
</tr>
</tbody>
</table>

**Binary operators** – \( \wedge \) conjunction (AND), \( \vee \) disjunction (or), \( \rightarrow \) implication (if-then)
\( a \land b \) – a and b
\( a \lor b \) – a or b
\( a \rightarrow b \) – a implies b, if a then b

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>( a \land b )</th>
<th>!a</th>
<th>( a \lor b )</th>
<th>!(a \land b)\</th>
<th>!(a \lor b)\</th>
<th>!a \lor b</th>
<th>!(a \lor b)</th>
<th>!(a \land b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>FALSE</td>
</tr>
<tr>
<td>TRUE</td>
<td>FALSE</td>
<td>FALSE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>FALSE</td>
<td>FALSE</td>
<td>FALSE</td>
</tr>
<tr>
<td>FALSE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>FALSE</td>
</tr>
<tr>
<td>FALSE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>TRUE</td>
</tr>
</tbody>
</table>

EQUIVALENCE

**Definition:** when 2 formulas \( F_1, F_2 \) have the SAME truth values for ALL combinations of input values we say that these formulas are EQUIVALENT, denoted \( F_1 \equiv F_2 \)

Most common equivalences
\( a \rightarrow b \equiv \!a \lor b \)
\( \!(a \land b) \equiv \!a \lor \!b \)
\( \!(a \lor b) \equiv \!a \land \!b \)

Equivalences are very useful; if \( F_1 \equiv F_2 \), can substitute one for another any time
Using in proofs: "it can be easily shown using truth tables that the above proposition is equivalent to ..."

One more operator - XOR - exclusive-or ⋆
\[ a \otimes b \equiv (a \lor b) \land (!a \lor !b) \]  
"at least one is true, and at least one is false"

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>a \otimes b</th>
<th>!a</th>
<th>!b</th>
<th>a \lor b</th>
<th>!a \lor !b</th>
<th>(a \lor b) \land (!a \lor !b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRUE</td>
<td>FALSE</td>
<td>TRUE</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>TRUE</td>
<td>TRUE</td>
<td>FALSE</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>FALSE</td>
<td>TRUE</td>
<td>TRUE</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>FALSE</td>
<td>FALSE</td>
<td>FALSE</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

Connection to boolean algebra

FALSE translates to 0, TRUE translates to 1 (or anything besides 0)  
\( \land \) translates to MULTIPLY, \( \lor \) translates to ADDITION

Translating English to propositional logic – very important skill

“unless” is connected to implication  
unless you give me $1M (G), I kill you (K)  
\( !G \rightarrow K \)

“but” means AND  
I love you (L) but I think you are fat (F)  
\( L \land F \)

“both C and D are liars” = “C is a liar” AND “D is a liar”