Reading (in textbook, unless stated otherwise):

- Section 8.3 middle of p. 508 (“definition of an equivalence relation”) through example 8.3.4
- Section 8.5 from begining (“partial order relations”) through example 8.5.5
- Section 6.1 from bottom of p. 384 (“partitions of sets”) to middle of p. 386 (up to “power sets”)

NOTE: please study examples carefully, there are many more of them than we possibly have time for in class

REVIEW

RELATIONS DEFINITION: Relation R is a set of pairs (a, b) where a, b ∈ D

NOTE: both a, b are from the same domain

Given a pair (a, b) in relation R, we can interchangeably write this fact the following 3 ways:

R(a, b) vs. (a,b) ∈ R vs. a R b (the last one preferred when R is a math symbol)

Examples:

- ≤ is a relation between numbers, "LEQ"
- ⊆ is a relation between sets, "SUBSET"
- substr is a relation between strings which is TRUE when one is a substring of another

NOTE: R can be viewed as a TRUTH SET for a corresponding predicate over 2 variables (a, b)

substr("di", "dina") is TRUE ≡ ("di", "dina") is in the substr relation

Example: Relation < over set D = {1,2,3,4}

WHAT ARE ALL THE PAIRS (x, y) IN D SUCH THAT x < y?
Answer: (1,2),(1,3),(1, 4),(2,3), (2,4) (3,4)

THREE PROPERTIES OF RELATIONS (designate a relationship as R)

R is SYMMETRIC: for all a,b in D, (a,b) ∈ R → (b,a) ∈ R (definition)

Examples:

- married IS SYMMETRIC: X IS married TO Y IFF Y IS married TO X
- ≤ is not symmetric: it is not true that whenever x ≤ y, then y ≤ x
**R is REFLEXIVE**: for all x in D : (x, x) ∈ R (definition)

**Examples**: 
- ≤ is reflexive for all x, it always the case that x ≤ x
- SUBSTR is not reflexive (assuming we've defined it so one string has to be shorter than the other)

**R is TRANSITIVE**: FOR ALL a, b, c in D, whenever (a,b) ∈ R and (b,c) ∈ R, then (a,c) ∈ R (definition)

**Examples**: 
- it is true that whenever a < b and b < c then a < c, so < is transitive
- it is true that whenever a ≤ b and b ≤ c then a ≤ c, so ≤ is transitive
- it is true that whenever ancestor(a,b) and ancestor(b,c) then ancestor(a, c); so ancestor is transitive
- it is NOT true that whenever loves(a,b) and loves(b,c) then loves(a, c); so loves is not transitive

**Visualizing** the three properties: Representing a relation as arrows among elements (dots) in the domain (blob)

```
  x --- y  \ (x,y) ∈ R
```

**FOURTH PROPERTY**

**R is ANTISYMMETRIC**: for all distinct a,b in D : (a,b) ∈ R → (b,a) ∉ R (definition)

**NOTE**: It is possible for a relation to be neither symmetric nor antisymmetric

**EXAMPLE**: Sometimes loves(a,b) AND loves(b,a) but other times loves(a,b) but not loves(b,a)

**NOTE**: It is also possible for a relation to be both symmetric and antisymmetric

**Example**: = (it's a special case)

**PARTIAL ORDER** – a relation that's reflexive, antisymmetric, transitive (3 conditions)

**Examples**: ≤ , ⊆ , substr, ancestor are all partial orders (see more in textbook)

**EQUIVALENCE RELATION** (definition) – a relation that's reflexive, symmetric, transitive (3 conditions)

**Examples**: =, sameLength, sameBirthday (see more in textbook)
Note: to prove that something is not a partial order or not an equivalence relation, enough to show that one of the three conditions is not satisfied

**PARTITION** – grouping of a set $S$ into non-empty subsets such that every element in $S$ is in exactly one of these subsets

**Definition:** A partition of $S$ is a set $S'$ of non-empty subsets of $S$ (meaning, $S' \subseteq P(S)$) such that:

1. They *cover* $S$ – the union of all sets in $S'$ equals $S$
2. They are pairwise disjoint – for any two sets in $S'$, their intersection is empty

We will be seeing partitions later in the course.