For next lecture:


**Reading (in textbook, unless stated otherwise):**

- Chapter 1.1 through example 1.1.1
- Chapter 2.1 up to “compound statements”

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**Logistics**

*Course Page:* [http://dgoldin.engr.uconn.edu/cse2500](http://dgoldin.engr.uconn.edu/cse2500)

All information can be found there

*Introduction to Discrete Systems* -- Here *Discrete* is a mathematical term, referring to values that are non-continuous

Continuous values are what you see in calculus, where we can take limits as the distance between numbers converges to 0

No calculus in this class!

**Values** are anything one can assign to a *variable*. Examples of types of values we will work with:

- integers
- booleans (there are 2)
- reals
- characters
- strings
- objects in general

**Variable names** – the book uses single letters, we are not limited to this, as in programs

**Sets** of these values will also be important.

ex: CSE2500students = set of cse 2500 students

ex: UCONNstudents = set of all uconn students

CSE2500students ⊆ UCONNstudents

**Proofs** – like a path through your brain

- starts with “given” facts
- consists of an argument that proceeds in steps (we will number the steps)
- ends with the desired conclusion (QED)

QED – *quod erat demonstratum*, Latin for “it has thus been demonstrated”

**EXAMPLE:** Ann, Bob, Colin
In addition to given facts, use **prior knowledge**:
- **DEFINITIONS** (EX: $X^2 = X \times X$, $<$
- **THEOREMS**
- **FACTS PROVEN EARLIER IN THE SAME PROOF**

**EXAMPLE**: PROVE THAT there exists a number that's greater than its square

The steps have to be VALID!

**EXAMPLE**: we have two facts
1. if it rains, prof. goldin carry an umbrella
2. prof. goldin is carrying an umbrella

Can we conclude that it's raining??

NO! That involves making an assumption that prof. Goldin only carries an umbrella when it rains
We do not make unwarranted assumptions in this class!

**Logic** gives us the language and the tools for valid reasoning

Create boolean variables aka **propositions**

RAINS = "it is raining"
CARRY = "prof. goldin is carrying an umbrella"

Propositions are always either TRUE or FALSE, there are no shades of gray in this course

Write fact 1 like this: RAINS → CARRY

Can we conclude that: if prof. goldin is not carrying an umbrella, it's not raining ??

**YES**

We will prove that RAINS → CARRY is **equivalent to** !CARRY → !RAINS ; i.e. they have same meaning
This actually works no matter what meaning the variables RAINS and CARRY have

**EXAMPLE**: city of liars

Proving algorithm correctness

Example of a **case-based proof**

Another type of **proof is by contradiction**

The first step is to **assume** the OPPOSITE of your desired conclusion,
Use this assumption as if it were a fact
The goal is to reach a **contradiction** (when something is both TRUE and FALSE)
This means your original assumption had to be wrong.

**Example: smallest integer**

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**Notes**

**Ann, Bob, Colin EXAMPLE**
GIVEN:

1. There are three Friends: Ann, Bob, Colin
2. One is a plumber, 1 a lawyer, 1 a blacksmith
3. Colin likes duck hunting
4. Ann's brother is a plumber
5. The plumber is an animal lover (would never hurt an animal)
6. The lawyer is an only child (has no brothers or sisters)

Question: Who is Who?

ANSWER: Colin is the lawyer, Ann is the blacksmith, Bob is the plumber

PROOF:
1. From fact 3,5 we conclude that COLIN NOT PLUMBER
2. from fact 2,4 we conclude that ANN IS NOT PLUMBER
   (make no assumptions)
3. from fact 1,2 and steps 1,2 we conclude that BOB IS PLUMBER
4. from facts 4,6 we conclude that ANN IS NOT LAWYER
5. from steps 2,4 we conclude that ANN IS BLACKSMITH
6. from facts 1,2 and steps 5,3 we conclude that COLIN IS LAWYER
QED

EXAMPLE: PROVE THAT there exists a number whose square is less than it

PROOF:
1. Consider 1/2
2. 1/2^2 = 1/4
3. 1/4 < 1/2
QED

EXAMPLE: city of liars

We have two houses, on the right and on the left.
We know that one contains TREASURE and another JASON.
There is a guy in front of the house, either honest (always correct answers) or a liar (never correct answers).
WE DON'T KNOW WHICH

Q: WHICH ONE QUESTION TO ASK HIM TO CHOOSE THE RIGHT HOUSE?

A: Which house would the other guy tell me has the treasure?
If he answer RIGHT, go LEFT; if he answers LEFT, go RIGHT (i.e. choose the OTHER house)

PROOF THAT THIS ALGORITHM WORKS:
(reason by cases)

CASE 1:
1.1 the guy is honest; the other guy is liar;
1.2 he will answer what the liar would say;
1.3 so choose the OTHER house
CASE 2:
1. the guy is a liar; the other one is honest;
2. he will NOT ANSWER what the honest one would say;
3. so choose the OTHER house

**Smallest Integer Example**

**TO PROVE:** THERE IS NO SMALLEST INTEGER

**PROOF:**
1. SUPPOSE there exists a SMALLEST INTEGER, call it X
2. consider $Y = X-1$;
3. $Y$ is an integer smaller than $X$
4. but $X$ is smallest!! CONTRADICTION
5. Therefore there does not exist a smallest integer

QED